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Comment on “Understanding the scalar meson $q\bar{q}$ nonet”

E. van Beveren[1], G. Rupp[2]

[1] Departamento de Física, Universidade de Coimbra, P-3000 Coimbra, Portugal (e-mail: eef@malaposta.fis.uc.pt)
[2] Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, Edifício Ciência, P-1096 Lisboa Codex, Portugal (e-mail: george@ajax.ist.utl.pt)

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Abstract. It is shown that the incomplete scalar meson nonets found by Törnqvist [1] and Törnqvist & Roos [2], employing a new version of the Helsinki unitarised quark model, should in fact be complete, including an as yet unconfirmed light K_0^* below 1 GeV (old κ) and the established $f_0(1500)$. A detailed comparison is presented with the predictions of the Nijmegen unitarised meson model, in which two complete scalar nonets show up below 1.5 GeV [3]. The reason for the flavour-nonet breaking found in [1,2] we argue to originate in the use of coupling constants for the three-meson vertex which are not independent of flavour. Also some statements made in [1] are critically reviewed.

A microscopic understanding of the scalar mesons remains a highly topical and unsettled issue in hadron spectroscopy. The apparent impossibility to group the experimentally established scalar states in complete nonets has triggered a lot of speculation on theoretical descriptions alternative to the standard $q\bar{q}$ configurations, including multiquark and gluonic states, and also two-meson bound states or molecules. Nevertheless, in [1], Nils A. Törnqvist presented a revised version of the Helsinki unitarised quark model (HUQM) (see also [4]), which describes the light scalars $f_0(980)$, $a_0(980)$, $f_0(1370)$, and $K_0^*(1430)$ as standard P-wave $q\bar{q}$ states, but with large components of two pseudoscalar mesons. Moreover, in [2], Törnqvist and Roos (TR) also found candidates for the $f_0(400\text{-}1200)$, the “good old” sigma meson, and the $a_0(1450)$ resonance, after a more thorough inspection of the complex energy plane. Crucial for these findings was the manifestation of a so-called resonance-doubling phenomenon, typical of some bare states coupled to S-wave decay channels with very large couplings. However, no light strange scalar was found in [1,2], and not even the by now established $f_0(1500)$, so that both the light scalar nonet below 1 GeV and the usual one in the region 1.3–1.5 GeV would remain incomplete. We are aware that the $f_0(1500)$ is often quoted as a serious candidate for a scalar glueball (see [5] for discussion and further references). Our own interpretation will be presented below. The actual values of the couplings employed by TR and also the number and location of decay channels were invoked to justify this strange breaking of the normal nonet pattern for colourless $q\bar{q}$ states.

In this Comment, we shall demonstrate that no such breaking should occur and that the findings of TR are due to the use of couplings constants which are *not* independent of flavour, since $SU(3)_{\text{flavour}}$ symmetry does not necessarily imply symmetry between flavour octets and singlets. In fact, about a decade earlier, we and our coworkers published a paper [3] in which we applied the Nijmegen unitarised meson model (NUMM) to the scalar mesons, and found two *complete* scalar nonets. For all members of these two nonets, there exist by now clear candidates in the Particle Data Group tables [6], with the exception of a light K_0^* below 1 GeV. However, also the existence of such a resonance has recently received increasing phenomenological and theoretical support [7–10]. Moreover, also modern hyperon-nucleon potentials based on meson exchange require a light K_0^* [11].

The NUMM is based upon much the same philosophy as the HUQM: mesons, both the stable ones and the resonances, are treated as $q\bar{q}$ systems coupled to open (real) and closed (virtual) meson-meson decay channels through the $^3P_0$ quark-pair-creation mechanism. The employed formalism is the coupled-channel Schrödinger equation, with some kinematically relativistic adjustments. This way, mesonic loops are automatically included in a non-perturbative and manifestly unitary framework. Contrary to the HUQM, however, a specific confining potential model has been chosen so as to obtain the so-called “bare” states of TR. Details of the NUMM can be found in [12] and [13], where the model was applied to heavy quarkonia and to all light and heavy pseudoscalar and vector mesons, respectively. The model parameters fitted there have been left *unaltered* in [3], so that our scalar-meson results are model predictions, and not the result of new fits as is the case in [1] and [2]. Another difference is our inclusion of all meson-meson coupled channels involving ground-state pseudoscalar and vector mesons, whereas TR limit themselves to pseudoscalars only. In fact, in the NUMM all three-meson vertices for any three meson nonets are related to one universal coupling constant, whereas in the HUQM, for different nonets, different couplings must be introduced.

The NUMM is formulated in such a way that it can be solved analytically, resulting in a non-perturbative multichannel scattering matrix, as a function of the total energy, the quantum numbers, the quark masses, the threshold masses of the two-meson channels, and the universal coupling constant g [14]. By insertion of the orbital and spin quantum numbers, this scattering matrix describes mesons with definite J^{PC}. The various flavours are obtained by insertion of the flavour quantum numbers and masses of the quarks, as well as the associated threshold masses of the two-meson systems to which they couple. One may then determine (numerically) the phase shift and/or cross section as a function of the total energy for any of the two-meson channels. The resulting cross sections each show an infinity of resonances, way beyond the energy domain where the model is intended to describe meson-meson scattering. For small values of g, these resonances are narrow. Moreover, the NUMM is constructed in such a way that the resonances are equidistant in the decoupling limit, corresponding to the radial and angular excitations of the harmonic-oscillator spectrum. This is, however, not the case for the universal value of g which fits experiment. To each resonance corresponds a complex singularity in energy, the position of which can easily be determined numerically, since we have an analytic expression for the scattering matrix at our disposal. In the decoupling limit, these singularities move towards the harmonic-oscillator states. However, there is not a unique relation between the latter states and the singularities. The trajectories of singularities depend on the way the decoupling is performed, either switching off the coupling to all two-meson channels at the same time, or dealing with each channel separately.

Now, for S-wave scattering, we find that the lowest singularity disappears to minus infinity imaginary part in the lower half of the complex energy plane, when the overall transition intensity g is turned off. But there exist other decoupling limits, for which the same singularity ends up on the harmonic-oscillator ground state, due to saddle points in the complex energy plane, which are functions of the coupling constants. This phenomenon we call *pole doubling*, which corresponds to the resonance doubling reported in [1,2]. However, we obtain it for *all* ground-state scalar mesons. The reason why it is only observed for S-wave scattering can be partly understood by using the effective-range expansion [15].

In Table 1, we show the model results of [1,2] and [3] for the real parts of the scalar resonances up to 1.5 GeV, together with the obvious experimental candidates.

Here, the "≈" signs indicate that these $q\bar{q}$ states are not pure due to the inevitable mixing of the isosinglets $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ and $s\bar{s}$ through the $K\bar{K}$ (and $K^*\bar{K}^*$ in [3]) channels, apart from the additional admixture of large meson-meson components for all resonances (see also the discussion on this point below). So the used $q\bar{q}$ designation is determined by the bare states the resonances are

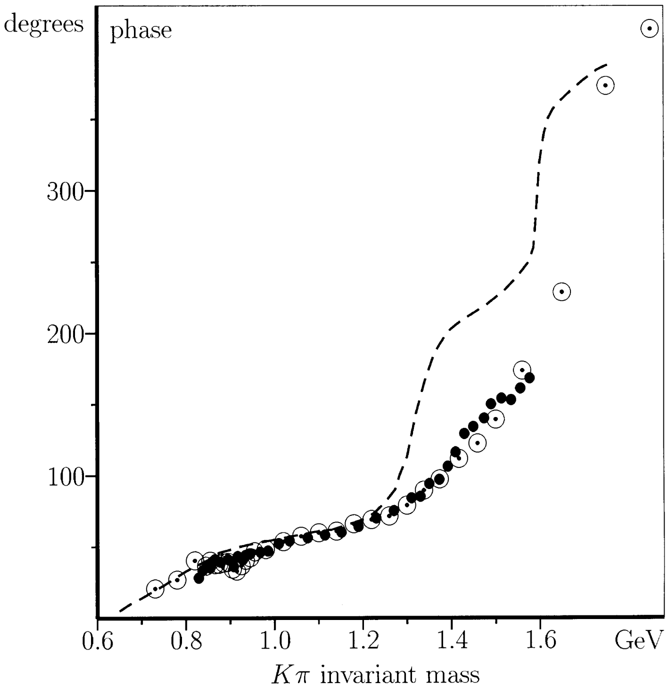

Fig. 1. Kaon pion $I=\frac{1}{2}$ S-wave phaseshifts. The data indicated by $\odot$ are taken from [18] and by $\bullet$ from [19]. The model results (dashed line) are taken from [3]

obtained from by turning on the overall coupling constant g. Moreover, "first" and "second" refer to the lower respectively higher pole of the pair of poles associated with the *same* ground-state bare $q\bar{q}$ state. As described above, in the NUMM the higher pole is the one that can be straightforwardly linked to the bare state by reducing the overall coupling, whereas the lower one escapes to $-i\infty$ in the limit $g \to 0$. From Table 1 we see that in [1,2] neither a light K_0^* state is found, nor the $f_0(1500)$, in contrast with [3]. Furthermore, the f_0 state around 1.3 GeV is interpreted as predominantly $s\bar{s}$ in [1,2] and as mainly $\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$ in [3].

Concerning the strange sector, the absence of resonance doubling is explained in [2] by arguing that only one important channel is open ($K\pi$), that it involves two unequal-mass mesons, and, most notably, that its coupling to $s\bar{d}$ is reduced by a factor $\sqrt{3/4}$ as compared to the coupling $s\bar{s}$–$K\bar{K}$. If the latter were true, it might indeed result in a pole too far away from the real axis to have any noticeable influence, since also we have found that these "image" poles tend to disappear to $-i\infty$ for decreasing coupling (see above). However, we obtain exactly the same coupling for these two cases (see Table 1 of [3] and Table 4 of [16]). In order to see where this discrepancy comes from, we inspect Table 1 of [1] and verify that these couplings are *not the same* for the various isospin multiplets. To this end, we take the squares of the numbers in each of the rows in the table and observe that they do not add up to the same result, since we find 3 for both the isotriplet and the isodoublet, 5 for the non-strange ($n\bar{n}$) isosinglet, and 4 for the $s\bar{s}$ isosinglet.

Table 1. Scalar-meson predictions and $q\bar{q}$ interpretations for the HUQM and NUMM, together with experimentally established states

	HUQM		NUMM		Experiment
	[1], [2]		[3]		[6]
Resonance	ReE_{pole}	$q\bar{q}$ configuration	ReE_{pole}	$q\bar{q}$ configuration	Mass
$\sigma/f_0(400–1200)$	470	$1^{st} \approx \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	470	$1^{st} \approx \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	400-1200
$S^*/f_0(980)$	1006	$1^{st} \approx s\bar{s}$	994	$1^{st} \approx s\bar{s}$	980 ± 10
$\delta/a_0(980)$	1094	$1^{st}\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	968	$1^{st}\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	983 ± 1
κ/K_0^*	-	-	727	$1^{st}\ s\bar{d}$	?
$f_0(1370)$	1214	$2^{nd} \approx s\bar{s}$	1300	$2^{nd} \approx \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	1200–1500
$f_0(1500)$	-	-	1500	$2^{nd} \approx s\bar{s}$	1500 ± 10
$a_0(1450)$	1592	$2^{nd}\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	1300	$2^{nd}\ \frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	1474 ± 19
$K_0^*(1430)$	1450	$1^{st}\ s\bar{d}$	1400	$2^{nd}\ s\bar{d}$	1429 ± 6

As a consequence, different mass splittings are generated, even if we hypothetically assume equal quark masses and equal thresholds for all flavours. In contrast, our couplings are truly flavour blind, as can be easily verified from the above-mentioned tables. In a separate paper [16], we develop a general method to derive such couplings for all OZI-allowed decays of any $q\bar{q}$ configuration. But also the other arguments invoked in [2] to justify the absence of the resonance-doubling phenomenon in the strange sector do not convince us. In [15], as mentioned above, we have shown that resonance doubling is peculiar to S-wave scattering, provided the couplings are sufficiently large, and that threshold effects, albeit important, are not decisive, for nothing singular happens there [17]. Furthermore, the fact that the $K\pi$ channel involves two (highly) unequal masses is irrelevant for the K_0^*, since only below threshold (630 MeV) the pseudothresholds will start to exert a noticeable influence. Of course, the good fit to the S-wave $K\pi$ phase shifts from 0.8 to 1.5 GeV obtained in [1] (Fig. 4) seems to lend credibility to a resonance-free description in the region 0.8-1.2 GeV. However, one should realise that this fit involves 4 parameters and that precisely at the endpoints of the fitted energy interval there is an onset of quite serious deviations, giving rise to, for instance, a wrong scattering length. As a comparison, we show our phase-shift predictions, without any fit in the scalar-meson sector, for the energy interval 0.7-1.7 GeV, depicted in Fig. 1. We point out that the description in the region 0.7-1.2 GeV, exactly where we find a light K_0^*, is almost perfect, including the scattering length. Thereabove, significant deviations occur due to the too small imaginary parts of the poles we find in the region 1.3-1.7 GeV, leading to too abrupt jumps in our phase shifts. Nevertheless, we do agree with experiment on the number of resonances there and on the gross behaviour of the phases, without having performed a fit.

Coming now to the $f_0(1500)$, it is quite surprising that this state has not been found in [1,2]. For the $f_0(1370)$, which is interpreted by those authors as a predominantly $s\bar{s}$ state, has, according to their own numbers (see Table 1 of [1]), a smaller coupling to its dominant decay channel, i.e., $\sqrt{2}$ vs. $\sqrt{3}$. Moreover, in both cases at least one more important decay channel is open, with two equal-mass mesons. So there appears to be a contradiction with the arguments invoked by TR to justify the absence of the resonance-doubling phenomenon in the strange sector, since they do not observe it in what they claim to be the $u\bar{u}+d\bar{d}$ case, while they do so for the supposed $s\bar{s}$. But also for these two cases, the couplings in the referred table do not satisfy flavour independence, as the squares of the numbers in the $u\bar{u}+d\bar{d}$ row add up to 5 and those in the $s\bar{s}$ row to 4.

As a matter of fact, we believe that it is more natural to interpret the $f_0(1370)$ as mainly $u\bar{u}+d\bar{d}$ and the $f_0(1500)$ as mainly $s\bar{s}$, in agreement with our model findings and also compatible with the particle-data meson tables [6]. For the principal decay modes of the $f_0(1370)$ involve non-strange mesons, i.e., two and four pions, whereas those of the $f_0(1500)$ concern the η and η', which have a strange-quark content [6]). So, while the decay modes of both resonances involving η's have similar strength, the pionic ones of the $f_0(1500)$ seem to be relatively suppressed due to a smaller non-strange-quark content. The small $K\bar{K}$ partial decay width of the $f_0(1500)$ is often argued to be in conflict with an $s\bar{s}$ assignment. But it is at least qualitatively in agreement with a (predominantly) octet configuration for the $f_0(1500)$ [20]. For that purpose, one can also check the fourth line of Table 4 of [16], under octet isoscalars to dd and e.g. tt, to verify that the octet coupling to $K\bar{K}$ is only one-third of that to $\pi\pi$. Now this does not mean that we claim the $f_0(1500)$ to be a pure octet state in our model, which nevertheless would still be mainly (67%) $s\bar{s}$. *A forteriori,* we cannot even make a definite statement about the exact mixing angle of $q\bar{q}$ components that strongly decay into inevitably large meson-meson components. The problem is that our mechanism for isoscalar mixing is totally dynamical and non-perturbative, taking place via the two-meson channels involving the K and K^*. Since the $f_0(1500)$, just like the other f_0 states, shows up as a complex pole in an S-matrix, it is impossible, even makes no sense, to precisely determine the degree of mixing, which would only be meaningful for a bound state.

Even a perturbative determination of the mixing would be highly unreliable, in view of the very large unitarisation effects. Furthermore, we must be very careful, with our approach, in drawing quantitative conclusions on decay rates from coupling constants and phase space only, since these rates, or better, the partial cross sections, are the non-linear results of a coupled-channel formalism. Anyhow, with the available experimental accuracy, the data on $f_0(1500) \rightarrow K\bar{K}$ are compatible with our model. Moreover, the possible existence of a scalar glueball in the same energy range, which would then mix with the unitarised $q\bar{q}$ resonance, cannot even be completely excluded at this stage.

Finally, we would like to comment on two statements made in [1], namely: *"Why has the solution presented here not been found previously?"*, on page 659, and *"..., no one has tried to fit simultaneously the whole nonet, taking into account all the light pseudoscalar thresholds, putting in physically acceptable analyticity properties, etc."*, on page 660. We think to have made it clear by now that the solution to the scalar-meson puzzle given in [1] and [2], which amounts to a revised version of the model of [4], had already been presented by us and our co-authors in [3], with a larger class of decay channels accounted for and also *really* flavour-blind coupling constants.

In conclusion, we want to emphasise the importance of experimentally confirming a light K_0^*, in order to lend even more credibility to the interpretation of the light scalar mesons as simple $q\bar{q}$ states with naturally large two-meson admixtures. Such a confirmation would also demonstrate that, despite the resonance-doubling phenomenon, the respect of flavour independence, when calculating three-meson couplings, guarantees the preservation of the standard nonet pattern for mesons in a unitarised description.

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